Cambridge International Examinations
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

8317605478

ADDITIONAL MATHEMATICS **4037/23**

Paper 2 **October/November 2014**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

DC (SJF/SLM) 97559

CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The expression $f(x) = 3x^3 + 8x^2 - 33x + p$ has a factor of $x - 2$.

(i) Show that $p = 10$ and express $f(x)$ as a product of a linear factor and a quadratic factor. [4]

(ii) Hence solve the equation $f(x) = 0$. [2]

2 A committee of four is to be selected from 7 men and 5 women. Find the number of different committees that could be selected if

(i) there are no restrictions, [1]

(ii) there must be two male and two female members. [2]

A brother and sister, Ken and Betty, are among the 7 men and 5 women.

(iii) Find how many different committees of four could be selected so that there are two male and two female members which must include either Ken or Betty but not both. [4]

3 Points A and B have coordinates $(-2, 10)$ and $(4, 2)$ respectively. C is the mid-point of the line AB.

Point D is such that $\overrightarrow{CD} = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$.

(i) Find the coordinates of C and of D. [3]

(ii) Show that CD is perpendicular to AB. [3]

(iii) Find the area of triangle ABD. [2]

4 The profit $\$P$ made by a company in its nth year is modelled by

$$P = 1000\mathrm{e}^{an+b}.$$

In the first year the company made \$2000 profit.

(i) Show that $a + b = \ln 2$. [1]

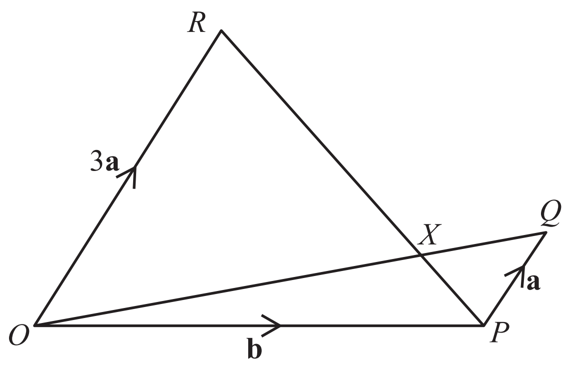
In the second year the company made \$3297 profit.

(ii) Find another linear equation connecting a and b. [2]

(iii) Solve the two equations from parts **(i)** and **(ii)** to find the value of a and of b. [2]

(iv) Using your values for a and b, find the profit in the 10th year. [2]

5



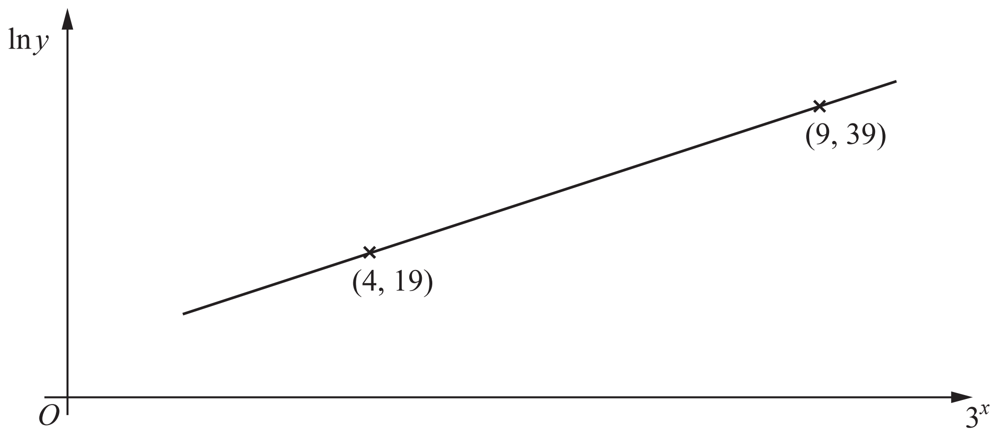
In the diagram $\overrightarrow{OP} = \mathbf{b}$, $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{OR} = 3\mathbf{a}$. The lines OQ and PR intersect at X.

(i) Given that $\overrightarrow{OX} = \mu\overrightarrow{OQ}$, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) Given that $\overrightarrow{RX} = \lambda\overrightarrow{RP}$, express $\overrightarrow{OX}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Hence find the value of μ and of λ and state the value of the ratio $\dfrac{RX}{XP}$. [3]

6 Variables x and y are such that, when $\ln y$ is plotted against 3^x, a straight line graph passing through (4, 19) and (9, 39) is obtained.



(i) Find the equation of this line in the form $\ln y = m3^x + c$, where m and c are constants to be found. [3]

(ii) Find y when $x = 0.5$. [2]

(iii) Find x when $y = 2000$. [3]

7 The functions f and g are defined for real values of x by

$$f(x) = \frac{2}{x} + 1 \text{ for } x > 1,$$

$$g(x) = x^2 + 2.$$

Find an expression for

(i) $f^{-1}(x)$, [2]

(ii) $gf(x)$, [2]

(iii) $fg(x)$. [2]

(iv) Show that $\text{ff}(x) = \dfrac{3x+2}{x+2}$ and solve $\text{ff}(x) = x$. [4]

8 A particle moving in a straight line passes through a fixed point O. The displacement, x metres, of the particle, t seconds after it passes through O, is given by $x = 5t - 3\cos 2t + 3$.

(i) Find expressions for the velocity and acceleration of the particle after t seconds. [3]

(ii) Find the maximum velocity of the particle and the value of t at which this first occurs. [3]

(iii) Find the value of t when the velocity of the particle is first equal to $2\,\text{m}\,\text{s}^{-1}$ and its acceleration at this time. [3]

9 **(i)** Determine the coordinates and nature of each of the two turning points on the curve $y = 4x + \dfrac{1}{x-2}$. [6]

(ii) Find the equation of the normal to the curve at the point (3, 13) and find the x-coordinate of the point where this normal cuts the curve again. [6]

10 **(i)** Prove that $\dfrac{1}{1-\cos x}+\dfrac{1}{1+\cos x}=2\operatorname{cosec}^2 x$. [3]

(ii) Hence solve the equation $\dfrac{1}{1-\cos x}+\dfrac{1}{1+\cos x}=8$ for $0° < x < 360°$. [4]